

February 1, 2008

UCRHEP-T121

EFFECTIVE LAGRANGIANS (for electroweak physics)*

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ABSTRACT

In this set of lectures I will present an elementary introduction to the uses of effective lagrangians in the electroweak interaction with emphasis in practical applications

* Lectures presented at the *IV Workshop on Particles and Fields of the Mexican Physical Society*. Mérida, Yucatán, México, Oct. 1993.

1. Introduction

I will describe the use of effective lagrangians and delineate the philosophy on which the method is based. I will then review the various theoretical details relevant to the construction of effective lagrangians insuring consistency with know experimental data.

Due to time limitations I will not be able to thoroughly cover the subject. To repair this deficiency I will refer the reader to the literature. Since this is intended to be a pedagogical review I have referred, whenever possible, to books and review articles instead of the original papers. The complete set of references can be found in the publications cited here.

1.1. *Motivation.*

Since the very first applications of field theory a very useful method in the description of certain phenomena has been the use of effective lagrangians. Classical examples are the Fermi theory of the weak interactions¹ and the BCS theory of superconductivity². A more recent example, equally successful, is the use of chiral lagrangians in the description of the strong interactions at low energies (for a review see Ref. 3).

All of the above theories were based on a set of well established experimental facts, such as the relevant particles involved in the phenomena under consideration, and the symmetries respected by them. It is understood that these models do not provide the ultimate, or most profound, description of the processes studied, but that there is a fundamental theory which coincides with the effective (phenomenological) model in the situations where the latter is applicable. For example, the four-Fermi interactions are now known to be the low energy remnants of the W and Z exchange interactions among fermions. An attractive feature of the effective lagrangian method is the minimal number of assumptions which are made concerning the underlying physics.

Another property of these theories is their limited range of applicability. In fact, the Fermi approach to the weak interactions is not to be applied above a certain energy, the BCS theory was not intended to describe the behaviour of metals at all temperatures, not are the chiral lagrangians a substitute for QCD. All of these effective theories have a range of validity, specified by a certain UV and IR scales, beyond which the model lagrangian should not be blindly applied for it will generate erroneous results. For example the Fermi current-current interaction predictions do not match the experimental results for energies comparable to the W mass; the chiral description of the strong interactions breaks down at energies of the order of $4\pi f_\pi$, etc.

A finite range of applicability may imply not only the presence of new interactions, but may signal a radical modification of the physical principles involved. A good example in this respect concerns classical hydrodynamics. This is a very good theory at scales larger than the interatomic distance, but when physics at smaller distances is studied, a radical modification of the manner in which the fluid is studied is required: quantum mechanics comes into play. Though this comment is probably not relevant for most of the processes to be studied in the foreseeable future, it should be kept in mind when thinking about the next great frontier, the gravitational interactions.

The procedure followed in all of the above examples in order to obtain the effective lagrangian is straightforward: determine the particles involved and the symmetries they are to obey, then construct the most general set of local interactions containing the corresponding fields constrained only by the said symmetries. (In practical calculations one determines the most important terms in this – in principle infinite – set of interactions neglecting the rest.) The lagrangian thus obtained has many dimensional parameters, some of these can be associated with the masses and other scales of the low energy physics. Other dimensional parameters, however, reflect the scale of the high energy physics underlying the effective lagrangian. It is because of this that current measurements can provide some information about the scale of new physics.

Effective lagrangians can be used in perturbative calculations, just as the “usual” lagrangians. As will be shown, the inevitable divergences which arise can be dealt with in the same way. The reason why a bad name has been sometimes associated with effective lagrangians is not related to their inability to deal with these divergences (since they can be treated in the same way as in the “usual” lagrangian case), but their alleged lack of predictability: these models contain an infinite number of coefficients, so that an infinite set of data points is apparently required in order to generate accurate predictions from such a model. The way out of this apparent deficiency is based on the fact that the terms in the effective lagrangian can be arranged according to a certain hierarchy such that the lower order terms generate the dominant contributions to any observable. Therefore, to any order in this hierarchy, there is only a finite number of operators which needs to be considered; one can also estimate the corrections produced by the terms which are neglected. Given an experimental result one can truncate the effective lagrangian calculation whenever the terms give contributions of the same order as the experimental uncertainty. Viewed in this light effective lagrangians are perfectly acceptable theories which do produce non-trivial predictions; moreover, they also provide the means of estimating the accuracy of such predictions.

Understanding how effective interactions are generated is simple. Suppose

the effective action for a model is obtained; assume now, for example, that a dimensional parameter is very large (alternatively suppose we are interested in energies much smaller than this scale). Expanding the effective action in inverse powers of this parameter generates an infinite set of local operators to be identified with the terms in the effective lagrangian. The full low energy content of the model is then described by this object. When the said parameter becomes large, several fields will acquire large masses; if the lagrangian with these fields removed is renormalizable then the decoupling theorem⁴ is applicable, and any observable can be expanded in inverse powers of the large dimensional parameter; in particular all effects of the heavy physics disappear when the dimensional parameter becomes infinite. In this case the above mentioned hierarchy in the effective lagrangian is determined by the expansion in inverse powers of the large scale; for example, the classification of the effective operators of dimension six with Standard Model fields was obtained in Refs. 5,6,7.

One can also consider the effect of having a large dimensionless parameter in the original lagrangian. Then the effective action can again be expanded in a series of local (effective) operators which determines then the effective lagrangian describing the low energy region of the model. When a dimensionless coupling is large some fields might also become heavy (note however that the *scale* of the corresponding masses is unaltered). In this case the decoupling scenario is not realized, and the hierarchy in the effective lagrangian is generated, in many physically interesting cases, by a derivative expansion.

The above two situations (to be denoted the *decoupling* and *non-decoupling* cases respectively) are quite different qualitatively and quantitatively. In the non-decoupling case the effects of a large (dimensionless) parameter do not disappear (and some observables may even diverge) in the limit when it becomes infinite. An example is the effects of a (formally) infinitely massive top quark in the Standard Model (with the vacuum expectation value fixed). This is not the case in the decoupling scenario, for example, all observable effects of an extra Z' vanish as $m_{Z'} \rightarrow \infty$.

The reason behind these different behaviours can be easily seen (qualitatively) by looking at the lagrangian. Setting a parameter to infinity imposes a constraint: that the operator that this parameter multiplies should vanish,^{#1} else there will be associated with this term an infinite energy. If the parameter corresponds to a mass then in the limit the field it multiplies will vanish; if the parameter is a mass term with the “wrong” sign (or a super-renormalizable coupling) then

^{#1} This discussion is purely classical, the quantum treatment of this problem lies beyond the scope of this course and I refer the reader to the literature⁴.

the constraint forces the fields multiplying the parameter to acquire a very large vacuum expectation value, and this implies that all fields getting a mass through this vacuum expectation value should vanish. In contrast, when a dimensionless parameter multiplying an interaction becomes infinite, the resulting constraint imposes a non-linear relationship among various fields and renders the low energy theory non-renormalizable.

To illustrate these possibilities consider the large Higgs mass limit of the Standard Model. The Higgs can be heavy either because the vacuum expectation value is big, or because the scalar self interaction coupling $\lambda \rightarrow \infty$. In the first case one must remember that the gauge boson and fermion masses also increase without bound, so that the low energy theory contains only photons and neutrinos. If in contrast we let $\lambda \rightarrow \infty$, the Higgs mass again becomes infinite (though the gauge boson masses are fixed), but the coupling between the would-be Goldstone bosons also becomes infinite. To elucidate this situation further consider the scalar doublet Φ and construct the matrix $\Omega = (\Phi, \tilde{\Phi})$, where $\tilde{\Phi} = i\sigma_2\Phi^*$, which satisfies $\Omega^\dagger\Omega = (\Phi^\dagger\Phi)\mathbf{1}$. I can then write

$$\Omega = \sqrt{\Phi^\dagger\Phi} U; \quad U^\dagger U = \mathbf{1}. \quad (1.1)$$

The whole of the Standard Model can be written in terms of Ω ; for example the scalar kinetic energy is $\text{tr}(D\Omega)^\dagger \cdot D\Omega$ where $D_\mu\Omega = \partial_\mu\Omega + \frac{i}{2}gW_\mu^a\sigma^a\Omega + \frac{i}{2}g'B_\mu\Omega\sigma_3$ ^{#2}. The scalar potential is

$$V = \lambda \left(\Phi^\dagger\Phi - \frac{1}{2}v^2 \right)^2 \quad (1.2)$$

so that the constraint imposed for large λ is $\Omega = vU/\sqrt{2}$ and the scalar sector becomes, in the roughest approximation, a non-linear sigma model. This is the so-called non-linear or chiral realization of the symmetry breaking in the Standard Model^{8,9,3,10,11,12}. The lowest order effective operators in this scenario have been obtained by several of the above references, I will use the conventions of Ref. 13. In the following I will refer to the effective lagrangian constructed from the gauge and fermion Standard-Model fields together with U , as the *chiral* case. Note that there are no physical scalar excitations in this scenario.

There are, of course, various other limits of interest. One can imagine letting a fermion mass increase by making a Yukawa coupling large¹⁴. In this case, just as in the previous situation, several interaction strength become large also. The

^{#2} This way of presenting things has many advantages, for example, it is immediate that the scalar sector has an $SU(2)_L \times SU(2)_R$ invariance broken only by the coupling to the $U(1)$ gauge field B_μ .

result is a kind of non-linear realization of the symmetry involving the fermions as well as the scalars. It might seem that these complications disappear, at least in the Standard Model, as soon as we imagine letting the Yukawa couplings of both elements of a doublet become large, but this is not so. First, since we are letting a dimensionless parameter become large, large fermion-scalar couplings are generated. Second, the theory without a doublet is not consistent, but in order to explain why I need to mention what an anomaly is.

The symmetries of a model are associated, via Noether's theorem^{1,15}, to a series of conserved quantities. It is then usually assumed that the corresponding symmetries are preserved after quantization; but this may not be the case. There are many situations (usually involving fermions) in which quantum corrections spoil certain symmetries. By this I mean that given an operator j_μ whose classical counterpart is conserved, it does not follow that the matrix elements of the operator $\partial \cdot j$ will vanish. They will of course be zero to $O(\hbar)$ since this is the classical result, but in certain situations the $O(\hbar)$ contributions do not vanish¹⁵. In this case the current j_μ and associated symmetry are labelled anomalous. If the anomalous current is also a gauge current then the theory is inconsistent¹⁶, at least within perturbation theory.

Returning to the previous discussion we imagine letting the (common) Yukawa coupling y of a Standard Model fermion doublet to be very large¹⁷. The doublet masses will then be very large, but the Standard Model without a doublet has an anomalous gauge symmetry and is therefore inconsistent. What explicit calculations show is that upon letting $y \rightarrow \infty$, a tower of CP violating scalar interactions is generated which build up to the so-called Wess-Zumino lagrangian¹⁸. This function of the scalar field is such that it generates the same anomaly as the apparently decoupled doublet. So the contributions from the light fermions plus the one produced by the WZ lagrangian render the theory non-anomalous. There are also other remnants of the heavy doublet. For example the contributions to the slope of the vacuum polarization of the vector bosons goes to a constant as y becomes infinite, and this slope (essentially the so-called S parameter) is measurable.

1.2. *Effective vs. renormalizable lagrangians.*

Many very successful field theories depend on a small number of unknown parameters. For example QED for photons and electrons has two parameters: the electron's mass and charge. The standard electroweak model has (ignoring topological terms) three gauge constants, two parameters in the scalar potential, nine masses and four parameters in the KM matrix: a total of 19 unknown parameters. The reason these models can get away with a finite number of parameters is tied to their renormalizability, which I describe next.

A theory is renormalizable if, in units where $\hbar = c = 1$, all terms in the lagrangian have mass dimensions ≤ 4 and if all propagators vanish at large energies. For theories with particles with spin ≥ 1 this last constraint requires some sort of gauge invariance (else the propagators either don't exist, as for massless particles, or else they go to an energy independent tensor at large energies, as is the case for massive particles).

The only difference between effective and renormalizable lagrangians lies in the allowed dimensions of the allowed terms: in effective theories there is no restriction whatsoever. Gauge invariance is still imposed for particles of spin ≥ 1 in effective theories: it is required by unitarity. It is a well known fact that theories with particles with spin ≥ 1 possess states of negative probability¹ and that the only known method of neutralizing the noxious effects of these states is based on a gauge principle (Gupta-Bleuler formalism, Fadeev-Popov technique, etc.)

If all allowed operators in an effective theory were equally important such a model would be useless (even writing down the lagrangian would be an impossible task). Fortunately the operators can be ordered so that, within the range of applicability of the model, higher order terms will generate small corrections to the results obtained using lower order terms. For example, if the heavy physics decouples, then all operators will be suppressed by the appropriate power of the heavy mass Λ and the effective lagrangian can be organized as a power series in $1/\Lambda$; this is also true for all observables. Thus when considering processes at energies $\ll \Lambda$, the higher dimensional operators will generate, in general, small corrections.

In the non-decoupling case the heavy physics is heavy due to the presence of a large dimensionless coupling. The resulting expansion is different: the decoupling theorem does not apply and, therefore, we can expect contributions to the effective lagrangians which do not vanish as the heavy mass increases. What can be done is then to present an expansion in powers of the external momenta or, equivalently, a derivative expansion⁸.

Given a lagrangian (effective or renormalizable) we have a set of undetermined parameters. When calculating the value of any observable the result will be a function of these parameters. Thus one can choose a (sufficiently large) number of observables and invert these relations; all parameters of the theory will then be expressed in terms of this set of observables, the "input data". The theory can now be used to predict the values of other observables in terms of the input data. As a technical detail I must point out that the calculation of observables cannot be done exactly and that some approximation technique is used.

A problem with the above program is that some calculations will be infinite. This is usually associated with integrals diverging in the UV limits of integration. The simplest example corresponds to the energy of the vacuum of a free bosonic

field: the field corresponds to an infinite set of harmonic oscillators¹ and so the vacuum energy diverges. To be able to handle such diverging quantities the model is regularized (for a clear discussion see Ref. 4). This means that the model under consideration is understood as the limit of a family of models such that for each member of the family the results are finite, while the divergences are recovered in the limit. Then, provided all relevant symmetries are obeyed by all members of the family, one can work with the lagrangians which give finite results, and take the limit at the end of any calculation (after all observables are expressed in terms of the input data).

For example one can require that the Fourier expansion of any field is cut off at a scale Λ_{UV} , and take the limit $\Lambda_{UV} \rightarrow \infty$ at the end of the calculation. This regularization prescription is problematic because one cannot in general preserve gauge invariance and Lorentz invariance. A more attractive possibility is to modify the kinetic term in the lagrangian. For example, in the case of a neutral scalar field ϕ the modification is

$$(\partial\phi)^2 \rightarrow (\partial_\mu\phi) [1 + (\Box/\Lambda_{UV}^2)^n] (\partial^\mu\phi) \quad (1.3)$$

where n is a sufficiently large integer. The problem with this method is that the propagator acquires a set of extraneous poles whose residues have the “wrong” sign; these represent states of negative norm. The scale of these unwanted states is Λ_{UV} so one has to prove that these poles leave no remnants when $\Lambda_{UV} \rightarrow \infty$.

A better method is to define the lagrangian in an arbitrary number of dimensions n , perform all calculations, and then take the limit $n \rightarrow 4$. This scheme is called dimensional regularization. In the first regularization techniques divergences are substituted by powers or logarithms of Λ_{UV} , in the last scheme they appear as terms proportional to $1/(n - 4)$ to some power.

So what is usually done is to first regularize the theory, then re-write all expressions in terms of the chosen set of observables (the input data). Though I will not prove it here, the expressions for all observables will then be finite in the limit where the regulator disappears (*i.e.* when the cutoff is set to infinity, the dimension of space-time is taken to four, etc.)⁴. For renormalizable lagrangians the substitution of lagrangian parameters for observables requires a finite set of operations, for the effective lagrangians the number of operations is (in principle) infinite.

For example, consider the graph below

representing a correction to the electron vacuum polarization in QED. Among other effects this graph shifts the pole in the electron propagator away from its original value m_0 : now the pole is at $m_0 + \alpha_0 m_0 \times (\text{divergent quantity}) = Z_m m_0$ ^{#3} (where α_0 is a parameter in the lagrangian describing the coupling of the electrons to the photons). It is then $m = m_0 Z_m$ that is measured in the laboratory as $9.1093897 \times 10^{-31}$ kg while m_0 by itself has no direct physical significance. Similarly $\alpha = Z_\alpha \alpha_0$ and $\alpha = 1/137.036$. When QED is dimensionally regularized, for example, the Z factors are expressed as Laurent series in $n - 4$; in fact they can be chosen as a power series in $1/(n - 4)$ with coefficients depending on α but independent of m . According to what I stated previously, when any physical prediction is written in terms of m and α (as opposed to m_0 and α_0) all the results are regulator independent and finite. The only point I glossed over pertains the possibility of rescaling the fields but, when S matrix elements are calculated, this is irrelevant.

1.3. *Symmetries and construction of the effective lagrangians.*

The ingredients required for the construction of an effective lagrangian are the relevant fields and the symmetries they are to obey. The requirement that the effective lagrangian be invariant under certain symmetries (and no others) can have important consequences. For example, a low energy description of the strong interactions should satisfy \mathcal{C} , \mathcal{P} and \mathcal{T} independently. But when the simplest lagrangian for the lowest meson octet is constructed, it is found to have two different operations associated with \mathcal{P} . This suggests that the simplest effective lagrangian is incomplete since it lacks the terms which break this group to the usual \mathbf{Z}_2 . This is in fact the case; the required terms are generated by the Wess-Zumino lagrangian¹⁸ which describes a wealth of phenomena previously unaccounted for, such as the decay of neutral pions into two photons. Thus, after one has constructed the most general set of operators with the chosen fields obeying the required symmetries,

^{#3} The correction is also proportional to m_0 due to chiral symmetry which prohibits the generation of a mass to all orders in perturbation theory. The quantity Z_m diverges as the regulators are removed, *i.e.*, as $\Lambda_{UV} \rightarrow \infty$, $n \rightarrow 4$, etc.

then one must verify that there are no spurious symmetries; if present this suggest possible terms have been omitted.

A prominent role among symmetries is occupied by gauge symmetries. This is so because, as mentioned above, they insure unitarity. It is however true that an arbitrary lagrangian can be understood as the unitary gauge limit of a gauge invariant lagrangian. This is shown by explicitly constructing the said gauge invariant lagrangian which contains in addition to the original fields a set of auxiliary fields¹⁹; such models are in general non-renormalizable. Thus it appears that gauge invariance is of no importance (since any theory can be thought as a gauge theory, albeit in the unitary gauge). There are, however, problems with this statement²⁰. First, the procedure by which a model is rendered gauge invariant does not fix the group (so that, for example, if we are given the Standard Model lagrangian in the unitary gauge we can turn it into an $SU(2) \times U(1)$ or a $U(1)^4$ gauge theory). Second, the gauge transformation properties of the matter fields are not fixed in this procedure. When the (experimentally supported) transformation properties of the matter fields and the values of the gauge couplings are imposed the (now gauge invariant) model acquires content, but then the requirement that it is gauge invariant is no longer trivial.

Once gauge invariance is imposed on an effective lagrangian the number of allowed operators is severely limited. For example²⁰, the coupling of the W bosons to the Z and photon is described, in the decoupling case to order $1/\Lambda^2$, by four parameters instead of the seven allowed by imposing only Lorentz invariance. Again to this order, the couplings among four vector fields are described by two parameters, etc.

1.4. *Equations of motion.*

It is often found that two effective operators differ only by terms that vanish when the classical equations of motion are assumed. In this case the S matrix cannot distinguish between them; this is trivial in the case of tree level diagrams involving one insertion of these operators, but is also true for loop graphs as I will now prove; I will restrict myself to the simple case of a scalar theory (for a complete discussion see Refs. 9,21). I will denote the fields by ϕ and the classical action by $S(\phi)$.

Suppose that we have two operators \mathcal{O} and \mathcal{O}' such that

$$\mathcal{O}' = \mathcal{O} + \int d^4x \mathcal{A} \frac{\delta S}{\delta \phi} \quad (1.4)$$

for some local quantity \mathcal{A} depending on the χ . The effective action is

$$S_{\text{eff}} = S + \int d^4x \left(\frac{\alpha}{\Lambda^2} \mathcal{O} + \frac{\alpha'}{\Lambda^2} \mathcal{O}' \right) + \dots; \quad (1.5)$$

the dots indicate higher dimensional operators. Let $S' = S + \int d^4x [(\alpha + \alpha')/\Lambda^2] \mathcal{O}$, then

$$\begin{aligned} S_{\text{eff}} &= S'(\phi) + \frac{\alpha'}{\Lambda^2} \int d^4x \mathcal{A} \frac{\delta S}{\delta \phi} + \dots \\ &= S'(\phi) + \frac{\alpha'}{\Lambda^2} \int d^4x \mathcal{A} \frac{\delta S'}{\delta \phi} + \dots \\ &= S'(\phi + \alpha' \mathcal{A}/\Lambda^2) + \dots \end{aligned} \quad (1.6)$$

Thus, to the order we are working, the effects of \mathcal{O}' are to replace $\alpha \rightarrow \alpha + \alpha'$ and $\phi \rightarrow \phi + \alpha' \mathcal{A}/\Lambda^2 = \Phi$. Using

$$\phi = \Phi - \frac{\alpha'}{\Lambda^2} \mathcal{A}(\Phi) + \dots \quad (1.7)$$

the effects of replacing $\phi \rightarrow \Phi$ appear only in external legs as in the following figure

where light lines denote ϕ propagators and heavy lines denote Φ propagators. Dots indicate external legs.

The S matrix is obtained by amputating the external legs and putting the resulting expression on the mass shell; graphically

But the terms in a Green's function with two or more Φ legs emanating from the same external point are regular when that external point is put on mass shell; that is

These regular terms, when multiplied by the corresponding inverse propagator, will vanish when the mass shell condition is imposed. It follows that only the terms linear in Φ in the expansion of ϕ will contribute to the S matrix. Thus, if near the mass shell $\phi = z\Phi + \dots$ for some constant z (the dots indicate terms with more Φ fields or terms which vanish on mass shell) then the effect of the operator \mathcal{O}' is then the replacement $\alpha \rightarrow \alpha + \alpha'$ and the appearance of the wave function renormalization factor z . This last quantity is unobservable: the precise same factor appears in the propagator (near the mass shell), so when the lines are amputated the z factors cancel out. I can then conclude that we can take \mathcal{O}' into account by the simple replacement $\alpha \rightarrow \alpha + \alpha'$.

2. Tree level applications.

The simplest application of effective lagrangians appear in tree level processes^{#4} which I will describe with an example. Consider the process $pe_R \rightarrow \nu_L X$ which can be generated at HERA^{22,23,24}. In the Standard Model this is a very suppressed reaction: it requires a helicity flip and will therefore be proportional to the electron mass divided by the CM energy. On the other hand there are several dimension six operators that can contribute to this process whose effects might be important. I will assume that the heavy physics is weakly coupled.

The relevant four-fermi operators are

$$\mathcal{O}_{\ell q} = (\bar{\ell}e)\epsilon(\bar{q}u), \quad \mathcal{O}_{qde} = (\bar{\ell}e)(\bar{d}q), \quad \mathcal{O}'_{\ell q} = (\bar{\ell}u)\epsilon(\bar{q}e), \quad (2.1)$$

where I have adopted the notation and conventions of Bücmüller and Wyler⁵: ℓ and q denote the left-handed lepton and quark doublets respectively, e , u and d denote the right-handed electron, up and down quark fields, and $\epsilon = i\sigma_2$. The graphs for the process at hand containing these operators are simply

The remaining operators containing Standard Model fields which contribute to the process at hand are

$$\mathcal{O}_{De} = (\bar{\ell}D_\mu e)(D^\mu \phi), \quad \mathcal{O}_{\bar{D}e} = (\overline{D_\mu \ell}e)(D^\mu \phi), \quad \mathcal{O}_{eW} = g\bar{\ell}\sigma^{\mu\nu}\sigma^I e\phi W_{\mu\nu}^I \quad (2.2)$$

where ϕ denotes the scalar doublet, D_μ the covariant derivative, σ^I the Pauli matrices, and $W_{\mu\nu}^I$ the $SU(2)_L$ gauge field strength tensor with g the corresponding coupling constant. These operators contribute via t -channel W exchange via the graphs

^{#4} It must be emphasized that this means tree level processes in the effective lagrangian, the effective operators summarize the low energy limit of loop graphs involving heavy particles.

The lagrangian is therefore

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{St.Model}} + \frac{1}{\Lambda^2} \sum_i \alpha_i \mathcal{O}_i \quad (2.3)$$

(the sum over i runs over the above six terms). I have kept only the operators of lowest dimension contributing at tree level, as they give the leading contributions.

There are some experimental restrictions on the coefficients α_i ⁵: from K and π decays it is known that

$$\alpha_{qde} \simeq 0 \quad \text{and} \quad \alpha'_{\ell q} \simeq 2\alpha_{\ell q} \quad (2.4)$$

which I will adopt; these conditions are assumed to be the result of some (unknown) constraint stemming from the underlying physics. With these restrictions the four-fermion interactions correspond to a tensor exchange,

$$\mathcal{O}_{\ell q} + 2\mathcal{O}'_{\ell q} \propto (\bar{\nu}_L \sigma_{\mu\nu} e_R) (\bar{d}_L \sigma^{\mu\nu} u_L) . \quad (2.5)$$

With (2.3) we can calculate the cross section for the process at hand. Note that the two types of operators (2.1) and (2.2) will not interfere due to helicity conservation at the quark vertex (provided quark masses are ignored). The result is

$$\begin{aligned} \frac{d\sigma_L}{dx dy} = & \frac{|\alpha_{\ell q}|^2 s}{32\pi\Lambda^4} x [(2-3y)^2 U + (2-y)^2 \bar{D}] \\ & + \frac{1}{32\pi} \left| \frac{g^2 v}{\sqrt{8}} V_{du}^{(KM)} \frac{c}{\Lambda^2} \right|^2 \left[\frac{x^2 y (1-y)}{(xy + m_w^2/s)^2} (U + \bar{D}) \right] \end{aligned} \quad (2.6)$$

where x and y are the usual scaling variables, $v = \sqrt{2}\langle\phi\rangle \simeq 256$ GeV, U and \bar{D} are the (x and y -dependent) quark distribution functions; I also defined $c = \alpha_{De} - \alpha_{\bar{D}e} + 8\alpha_{eW}/g$. I will show later that when the underlying physics is weakly coupled $c \sim 1/(16\pi^2)$ and $\alpha_{\ell q} \sim 1$.

Using (2.6) we can estimate the sensitivity of HERA to the scale Λ : the above cross section will generate 15 events per year at HERA provided $\Lambda \lesssim 315$ GeV.

This result will be weakened for polarizations below 92%; in this case the statistical significance of the right-handed electron signal must be considered. Suppose that we have a beam of polarization \mathcal{P} , where $\mathcal{P} = 1$ implies pure right-handed electrons. Let σ_{SM} be the Standard Model contribution to $e_L p \rightarrow \nu_L X$ via t -channel

W exchange; a simple calculation shows that^{#5}

$$\sigma_{SM} = \frac{g^4}{8\pi s} \int_0^1 dx dy \frac{xU + x(1-y)^2 \bar{D}}{(xy + m_W^2/s)^2} \simeq 5.67 \times 10^{-7} \text{ GeV}^{-2}, \quad (\sqrt{s} = 292 \text{ GeV}) \quad (2.7)$$

The number of signal events is $N_{\text{signal}} = \mathcal{P} \sigma_L \mathbb{L}$, where \mathbb{L} is the luminosity ($\mathbb{L} = 4 \times 10^9 \text{ GeV}^2/\text{year}$ for HERA); the number of background events is $N_{\text{bckgnd}} = (1 - \mathcal{P}) \sigma_{SM} \mathbb{L}$; the condition for the signal to be statistically significant is

$$N_{\text{signal}} > \sqrt{N_{\text{bckgnd}} + N_{\text{signal}}}, \quad (2.8)$$

which determines the sensitivity to Λ given \mathcal{P} . This condition generates the following graph

where I assumed $|\alpha_{\ell q}| = 0.44$. This plot gives the maximum sensitivity to Λ for a given polarization. For $\mathcal{P} \leq 0.9$ the curve is generated by (2.8), for $\mathcal{P} > 0.9$ the curve corresponds to 15 events per year at HERA. For realistic values of \mathcal{P} the process is no longer rate dominated.

^{#5} There is a large class of operators which also modify the couplings of the W to the left-handed weak currents, as well as shifting the W mass from it's standard model value. I have not included these contributions in σ_{SM} since they represent but small corrections.

2.1. S, T, U .

Some of the best measured quantities which are sensitive to new physics are the oblique parameters S , T and U ²⁵ obtained from the vacuum polarization tensors for the W and Z bosons. In terms of the $SU(2) \times U(1)$ eigenstates

$$\begin{aligned} S &= -16\pi\Pi'_{3B}(0), \\ T &= \frac{4\pi}{(s_w m_w)^2} [\Pi_{11}(0) - \Pi_{33}(0)]; \\ U &= +16\pi [\Pi'_{11}(0) - \Pi'_{33}(0)]; \end{aligned} \quad (2.9)$$

where the indices 1 and 3 refer to $SU(2)$, the subindex B refers to $U(1)$. The functions Π are the transverse part of the vacuum polarization tensors, they are functions of p^2 ; the prime indicates a derivative with respect to p^2 .

The Standard Model contributions have been studied extensively in the literature; concerning the possible contributions from new physics I will use an effective lagrangian parametrization and consider the operators which have two gauge bosons, no fermions and any number of scalars. For the decoupling case these are⁵

$$\begin{aligned} \mathcal{O}_{\phi W} &= \frac{1}{2} (\phi^\dagger \phi) (W_{\mu\nu}^I)^2; & \mathcal{O}_{\phi \tilde{W}} &= \frac{1}{2} (\phi^\dagger \phi) (W_{\mu\nu}^I \tilde{W}_{\mu\nu}^I); \\ \mathcal{O}_{\phi B} &= \frac{1}{2} (\phi^\dagger \phi) (B_{\mu\nu})^2; & \mathcal{O}_{\phi \tilde{B}} &= \frac{1}{2} (\phi^\dagger \phi) (B_{\mu\nu} \tilde{B}_{\mu\nu}); \\ \mathcal{O}_{WB} &= (\phi^\dagger \sigma_I \phi) W_{\mu\nu}^I B^{\mu\nu}; & \mathcal{O}_{\tilde{W}B} &= (\phi^\dagger \sigma_I \phi) \tilde{W}_{\mu\nu}^I B^{\mu\nu}; \\ \mathcal{O}_\phi^{(1)} &= (\phi^\dagger \phi) |D_\mu \phi|^2; & \mathcal{O}_\phi^{(3)} &= |\phi^\dagger D_\mu \phi|^2; \end{aligned} \quad (2.10)$$

so that the effective lagrangian is

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{St.Model}} + \sum_i \frac{\alpha_i}{\Lambda^2} \mathcal{O}_i \quad (2.11)$$

where the sum over i runs over the above operators. I then find

$$S = \frac{2v^2}{\pi\Lambda^2} \left(\frac{16\pi^2 \alpha_{WB}}{gg'} \right); \quad T = -\frac{16\pi v^2}{s_w^2 \Lambda^2} \alpha_\phi^{(3)}; \quad U = 0. \quad (2.12)$$

For the chiral case the relevant effective lagrangian is¹³

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{\beta_1 g^2 v^2}{4} \left(\text{tr} \left\{ \sigma_3 U^\dagger D_\mu U \right\} \right)^2 + \frac{\alpha_1 g g'}{2} B^{\mu\nu} W_{\mu\nu}^I \text{tr} \left\{ U^\dagger \sigma_3 U \sigma_I \right\} \\ &+ \frac{\alpha_8 g^2}{4} \left(W_{\mu\nu}^I \text{tr} \left\{ U^\dagger \sigma_3 U \sigma_I \right\} \right)^2; \end{aligned} \quad (2.13)$$

whence

$$S = -\frac{1}{\pi} (16\pi^2 \alpha_1); \quad T = \frac{1}{2\pi s_w^2} (16\pi^2 \beta_1); \quad U = -\frac{1}{\pi} (16\pi^2 \alpha_8). \quad (2.14)$$

I will prove later that the natural size for the coefficients $\alpha_{1,8}$ is $1/(4\pi)^2$; the natural magnitude of β_1 is $\lesssim 1$ though experimental constraints requires $\beta_1 \lesssim 1/(16\pi^2)$. Comparing these two sets of expressions I can conclude that if the operators of dimension six in the effective lagrangian dominate (for the decoupling case) then I expect $U \simeq 0$; in contrast, for the chiral, case $U \sim O(0.1)$.

The operators in (2.10) are generated via loops in the heavy theory, except $\mathcal{O}_\phi^{(1,3)}$. Moreover each W will also be accompanied by a g and each B by a g' . Then $\alpha_{WB} \sim gg'/16\pi^2$ and $\alpha_\phi^{(3)} \sim 1$. Thus I expect $|S| \sim 2v^2/\pi\Lambda^2$ and $|T| \sim 16\pi v^2/(s_w^2 \Lambda^2)$. For $\Lambda \sim 1$ TeV I get $|S| \sim 0.15$; using $|T| \lesssim 1$ I get $\Lambda \gtrsim 10$ TeV

But there will be, in the decoupling case, contributions to S , T and U from tree-level-generated dimension eight operators. Then, for example,

$$S \sim \left[\underbrace{\left(\frac{v^2}{16\pi^2 \Lambda^2} \right)}_{\text{dim}=6, 1 \text{ loop}} + \underbrace{\left(\frac{v^4}{\Lambda^4} \right)}_{\text{dim}=8, \text{ tree level}} \right] \times 100 \quad (2.15)$$

so that the dimension eight operators dominate if $\Lambda < 4\pi v \sim 3$ TeV.

For the parameter U there are then two cases

$$U \simeq \begin{cases} 0 & \text{for } \Lambda > 4\pi v \\ 0.1 & \text{for } \Lambda < 4\pi v \end{cases} \quad (2.16)$$

The second cases comprising the both the chiral case and the decoupling case when the dimension-eight operators are important. The U parameter would be then very useful in distinguishing among these possibilities. As of now the measurements are to $O(1)$ but an improvement by a factor ~ 10 (and into the interesting region) are to be expected from Fermilab and LEP in the near future.

3. Properties of the effective lagrangians

3.1. *Magnitude of the coefficients.*

When considering an effective operator it is important to have quantitative estimates to be able to predict the order of magnitude of its coefficients. This allows for the determination of the dominant contributions to any given observable as well as its sensitivity to new physics effects. There are two cases of interest depending on whether the underlying physics decouples or not.

Loops vs. tree level contributions.

For the case where the underlying theory is weakly coupled this implies determining whether the operator in question is generated at tree level by the heavy dynamics or whether it is loop generated; in the second case, there will be a suppression of $\sim 1/16\pi^2$ in the coefficient. In addition there will be other suppression factors due to the presence of weak ($\lesssim 1$) coupling constants.

I will now describe how to determine whether an operator is generated at tree level or at one loop. I will assume that the theory underlying the Standard Model is a gauge theory; the corresponding gauge indices are a, b , etc. The gauge group, of course, contains the Standard Model gauge group, whose indices will be denoted I, J , etc.

Consider for example

$$\mathcal{O}_W = \epsilon_{IJK} W_{\mu\nu}^I W_{\nu\lambda}^J W_{\lambda\mu}^K. \quad (3.1)$$

If this operator is generated at tree level there must be a graph of the form

This is so because gauge invariance in the light degrees of freedom requires the full non-Abelian field tensor to appear in \mathcal{O}_W which therefore contains a term with six external light legs. The only tree-level graph which generates such a term in a Yang-Mills theory is then the one depicted above.

Note however, that $f_{bIJ} = 0$ when b is the index of a heavy gauge boson. This is true because the light vector bosons are the gauge bosons of a bona-fide gauge theory, hence the commutator of two generators with light indices will give a linear

combination of generators all with light indices. Hence the structure constants with two light and one heavy indices vanish. It follows that \mathcal{O}_W can only be generated by loop graphs of the type.

As another example consider the four fermion operator

$$\mathcal{O} = (\bar{\psi}_1 \gamma_\mu \psi_2) (\bar{\psi}_3 \gamma^\mu \psi_4) \quad (3.2)$$

which can be generated by a heavy gauge boson exchange as in the following graph

All operators appearing in Ref. 5 can be analyzed in this way. I refer the reader to Refs. 26,27 for a detailed discussion.

Strongly coupled case

In order to determine the order of magnitude of the coefficients of the effective operators it is natural to require that the radiative corrections to the coefficient of an operator be at most as large as its tree level value.

Consider a theory with scalar fields ϕ and fermionic fields ψ and gauge bosons W . Then the relevant vertices have the symbolic form

$$\Lambda^4 \lambda (\phi/\Lambda_\phi)^A (\psi/\Lambda_\psi^{3/2})^B (p/\Lambda)^C (gW/\Lambda)^D, \quad (3.3)$$

where p represents a derivative, Λ is a UV cutoff, λ is a coupling constant, and the other scales, $\Lambda_{\phi,\psi}$, are to be determined. Since Λ is associated with the momentum scale I divide p (a generic momentum) by this scale; since gauge fields appear only in covariant derivatives, they are divided by the same scale. The quantities A , B , C and D are assumed to be integers. Since the W fields appear always inside a covariant derivative it is sufficient to consider vertices with $D = 0$.

Now consider a graph with V vertices which generates an L loop correction to (3.3). This contribution will be of the same order provided (I replace all loop moment by Λ since we are interested only in an order of magnitude estimate)

$$1 \sim (\Lambda^4 \lambda)^{V-1} \Lambda_\phi^{A-\sum A_i} \Lambda_\psi^{3(B-\sum B_i)/2} \Lambda^{C-\sum C_i} \Lambda^{\sum C_i - C + 4L - 2I_b - I_f} (4\pi)^{-2L}, \quad (3.4)$$

where i labels the vertices in the graph and I_f (I_b) is the number of internal fermion (boson) propagators. Using the relations $\sum A_i = A + 2I_b$ and $\sum B_i = B + 2I_f$ (3.4) becomes

$$(16\pi^2 \lambda)^{-L} \left(\frac{\lambda \Lambda^2}{\Lambda_\phi^2} \right)^{I_b} \left(\frac{\lambda \Lambda^3}{\Lambda_\phi^3} \right)^{I_f} \sim 1; \quad (3.5)$$

this requires

$$\lambda \sim \frac{1}{16\pi^2}; \quad \Lambda_\phi \sim \frac{1}{4\pi} \Lambda; \quad \Lambda_\psi \sim \frac{1}{(4\pi)^{2/3}} \Lambda. \quad (3.6)$$

Substituting back into (3.3), and using the fact that gauge bosons appear always in a covariant derivative denoted by \mathcal{D} , yields

$$\frac{\Lambda^4}{(4\pi)^{2-A-B}} \left(\frac{\phi}{\Lambda} \right)^A \left(\frac{\psi}{\Lambda^{3/2}} \right)^B \left(\frac{\mathcal{D}}{\Lambda} \right)^C. \quad (3.7)$$

As a first application of this result consider the corrections to the vector boson masses. For this take $C + D = 2$, $A = B = 0$, then $M^2 \sim \Lambda^4 \lambda g^2 / \Lambda^2 = (\Lambda g / 4\pi)^2 = (g \Lambda_\phi)^2$. From the expression for the vector boson masses in a spontaneously broken gauge theory it follows that $\Lambda_\phi = v$ where v is to be identified with the vacuum expectation value of the scalars in the Standard Model. Then

$$\Lambda_\phi = v \simeq 246 \text{ GeV}, \quad \Lambda = 4\pi v \simeq 3 \text{ TeV} \quad \Lambda_\psi = (4\pi)^{1/3} v \simeq 572 \text{ GeV} \quad (3.8)$$

For the operator \mathcal{O}_W I must take $C + D = 6$ and $A = B = 0$ whence

$$\alpha_W \sim \frac{1}{\Lambda^2} \frac{g^3}{16\pi^2} \quad (3.9)$$

which is the same estimate as was obtained in the decoupling case.

3.2. *Many particles.*

In this subsection I wish to consider the situation where there are operators which are loop generated (when the heavy physics is weakly interacting), but when there are very many graphs contributing to such operators. In particular, what would happen if there were ~ 160 graphs adding coherently in their contributions to the said operator?

For the vector bosons we would expect corrections $\sim 100\%$ to their masses. For the scalar masses the contribution of N graphs of the type

(where the particles in the loop have a mass $\sim \Lambda$) will generate corrections $\sim N\Lambda^2/16\pi^2$ to the scalar's mass. Thus if there are ~ 160 such contributions adding coherently the masses of these scalars become $O(\Lambda)$; the corresponding fields are in fact not a part of the low energy lagrangian. If there are many contributions to the loop generated operators then one should expect very strong deviations from the naive low energy lagrangian: all (unprotected) scalar fields disappear from \mathcal{L}_{eff} and the protected masses get very large corrections.

3.3. *Gauge invariance.*

When considering effective theories it is often assumed that the gauge invariance present in the Standard Model is in fact a low energy symmetry broken at higher energies. I will argue that this assumption is inconsistent with the lightness of the gauge boson masses and with the experimental result that the gauge coupling constants are all small³⁰.

Consider a theory with vector bosons and fermions; the vector bosons have a mass of order M and propagator $(g_{\mu\nu} - p_\mu p_\nu/M^2)/(p^2 - M^2 + i\epsilon)$. I now consider the radiative corrections to the triple vector boson couplings and to the fermion-anti fermion-gauge boson couplings. The vertices of interest are

where the factor of p denotes a derivative (I will not need a more precise description of the vertices). I then obtain the following estimates

$$\sim \frac{1}{16\pi^2 M^4} g_A^2 \Lambda^6 \sim M^2$$

$$\sim \frac{1}{16\pi^2 M^4} g_f^3 \Lambda^2 \sim g_f$$

From these results it follows that

$$\frac{g_A}{4\pi} \sim \frac{M^3}{\Lambda^3}, \quad \frac{g_f}{4\pi} \sim \frac{M}{\Lambda}. \quad (3.10)$$

Thus, if as is the case for the Standard Model, $g_A \sim g_f$, then

$$M \sim \Lambda; \quad g_A, g_f \sim 4\pi \quad (3.11)$$

This implies that radiative corrections to the gauge boson masses will shift them out of the low energy theory; moreover, the corrections to the coupling constants are so big as to render the theory strongly interacting.

This argument strongly supports the claim that gauge invariance cannot be broken “softly” by higher dimensional operators.

3.4. *Blind directions.*

Another consideration relevant for quantitative estimates is the possible presence of “blind directions”³¹: these are operators to which we have no experimental sensitivity since they affect quantities which are precisely measured only at the one loop level (or beyond). An example is the operator

$$\mathcal{O}_W = \epsilon_{IJK} W_{\mu\nu}^I W_{\nu\lambda}^J W_{\lambda\mu}^K, \quad (3.12)$$

which is to be contrasted with the operator $\mathcal{O}_{WB} = (\phi^\dagger \sigma_I \phi) W_{\mu\nu}^I B^{\mu\nu}$ which contributes to the oblique S parameter and is well measured.

It has been assumed that the coefficients of these two operators are of the same order as it appears difficult to suppress one with respect to the other without fine tuning. This is in fact not the case: it is easy to generate models where there is such a natural suppression, so that the assumption that blind directions can be estimated based on the “sighed” directions remains an additional assumption to be tested by experiment.

Consider first the following toy model consisting of a light scalar field ϕ interacting with two heavy fermions ψ_a ($a = 1, 2$). The lagrangian is

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{6}\sigma\phi^3 - \frac{1}{24}\lambda\phi^4}_{\text{light sector}} + \underbrace{\sum_{a=1}^2 \bar{\psi}_a (i \not{\partial} - M + (-)^a g\phi) \psi}_{\text{heavy sector}} \quad (3.13)$$

the Feynman rules for this model are

from which the graphs

are seen to have a coefficient $[(-1)^a g]^n$. Since the contribution to the n scalar point function is the sum over $a = 1, 2$ it is clear that the contributions with an odd number of scalar legs cancel.

If low energy ($\ll M$) experiments are sensitive to, for example, ϕ^5 , but not to ϕ^6 (which is then a blind direction), there would be no experimental indication of the heavy sector. The null results could be interpreted as Λ being astronomical raising strong objections for spending any resources in measuring the effects of ϕ^6 , while in fact a new generation of experiments could very well uncover the presence of the heavy fermions.

The same results can be obtained in the Standard Model by considering two vector-like fermions of almost degenerated masses with opposite hypercharges. Then there are no contributions generated to any operator with an odd number of B legs, in particular \mathcal{O}_{WB} is not generated, while the contributions to \mathcal{O}_W from the fermions add up and so the coefficient is $\sim g^3/16\pi^2$

3.5. *Equations of motion and orders of magnitude.*

It is easy to find situations where two operators are equivalent in the sense that

$$\mathcal{O} - \mathcal{O}' = \int d^4x \mathcal{A}(\phi) \frac{\delta S}{\delta \phi} \quad (3.14)$$

where \mathcal{O} is tree level generated but \mathcal{O}' is generated at the one loop level. It is then quantitatively important *not* to eliminate \mathcal{O} in favor of \mathcal{O}' since the ensuing estimates for the coefficients can be wrong by several orders of magnitude.

4. Effective lagrangians at one loop.

I will consider now the one loop contributions generated by effective lagrangians. In many situations these contributions are negligible. This is usually so whenever the effective operator in a graph is loop generated: such graphs are in fact the low energy limit of certain two loop diagrams in the full theory.

There are however situations in which the effective-operator one loop contributions are important. Firstly when considering high precision data involving tree-level effective operators (whose coefficients do not have a loop suppression factor). But other situations can be imagined, for example it is possible for certain processes to be forbidden at tree level (with or without the presence of effective operators) and for the loop contributions without effective operators to be suppressed. In this case the loops involving effective operators can be of importance.

An example of this last case is the decays of the CP-odd scalar a_o present in the two scalar doublet extension of the Standard Model³². I assume flavor changing neutral currents are suppressed by imposing a discrete symmetry, and consider the process $a_o \rightarrow \gamma\gamma$ generated by (only) quark loops.

Since the couplings are^{#6} $a_o \bar{t}t \propto m_{\text{top}}/\tan\beta$ and $a_o \bar{b}b \propto m_{\text{bottom}}\tan\beta$; then for $\tan\beta \gg 1$, $B(a_o \rightarrow \gamma\gamma) \simeq 0$. This suggest we study the effects of higher dimensional operators for this model (generated by physics beyond the two doublet model)³³. When this is done it is easy to verify that the discrete symmetries forbids all $a_o\gamma\gamma$ couplings in operators of dimension ≤ 6 . Since there are no tree level contributions the loop graphs will be finite, as we will see. Moreover these loop contributions need not be small in the $\tan\beta \gg 1$ limit^{#7}

Finally there is one last reason why loop calculations are important: I have stated above that effective theories are actually renormalizable and the verification of this claim with explicit computations is of importance.

^{#6} $\tan\beta$ denote the ration of the vacuum expectation values of the scalar doublets in this model.

^{#7} The effective lagrangian contributions in this example can also dominate in the a_o creation in photon-photon collisions.

4.1. *Dimensional regularization.*

I will present here a very brief introduction to dimensional regularization; for a thorough review see Ref. 4. I will limit myself to zero spin bosonic theories (though no new concepts are involved when this is extended to vector bosons and fermions, the only complication concerns the proper definition of γ_5). As mentioned previously the idea is simply to define everything in n dimensions and letting $n = 4 - \epsilon$ with the understanding that $\epsilon \rightarrow 0$ at the end of the computation.

The first ingredient is Feynman's trick for combining denominators:

$$\prod_i \frac{1}{A_i^{\alpha_i}} = \frac{\Gamma(\sum_i \alpha_i)}{\prod_i \Gamma(\alpha_i)} \int_0^1 \prod_i dx_i \delta\left(1 - \sum_i x_i\right) \frac{\prod_i x_i^{\alpha_i-1}}{(\sum_i A_i x_i)^{\sum_i \alpha_i}} \quad (4.1)$$

where all the A_i are assumed to have the same-sign imaginary parts. For example, $1/(A_1 A_2) = \int_0^1 dx [xA_1 + (1-x)A_2]^{-2}$.

As the second ingredient I need the Feynman rules. To avoid unnecessary complications I will consider a simple theory whose lagrangian is

$$\mathcal{L} = \frac{1}{2} (\partial\phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{1}{4!} \lambda \phi^4 \quad (4.2)$$

then the Feynman rules are

$$\frac{i}{p^2 - m^2 + i\epsilon}$$

$$-i\lambda$$

The theory is regulated by defining it in n dimensions (the metric has the form $\text{diag}(1, -1, -1, \dots)$.) The recipe to generate the Green's functions in a (regulated) perturbative expansion is to draw all possible graphs, each vertex and line given as above with momentum conservation imposed in each vertex. One finds that some momenta are not fixed by this condition, these are called loop momenta. Any such loop momentum l must be integrated over with measure $d^n l / (2\pi)^n$. Finally, to each graph one has to associate a symmetry factor $1/w!$ for each group of w lines which can be permuted without altering the graph in any way. The number of vertices determines the power of λ and, therefore, the order in perturbation to which we are working.

For example, the diagram

(with the slashes in the external legs indicating that the corresponding propagators are omitted) corresponds to the following integral

$$I_n = \frac{1}{2}(-i\lambda) \int \frac{d^n k}{(2\pi)^n} \frac{i}{(k+p)^2 - m^2 + i0} \frac{i}{k^2 - m^2 + i0} \quad (4.3)$$

imagining for the moment that n is an integer. Next I combine denominators to obtain (I define $d_n k = d^n k / (2\pi)^n$)

$$\begin{aligned}
I_n &= \frac{1}{2}(+i\lambda) \int d_n k \int_0^x dx \frac{1}{\{x[(k+p)^2 - m_i^2 0] + (1-x)(k^2 - m^2 + i0)\}^2} \\
&= \frac{i\lambda}{2} \int d_n k \int_0^1 dx \frac{1}{(k^2 + 2xk \cdot p + xp^2 - m^2 + i0)^2} \\
&= \frac{i\lambda}{2} \int d_n k \int_0^1 dx \frac{1}{[(k+xp)^2 - \xi]^2}; \quad \xi = m^2 - x(1-x)p^2 - i0. \\
&= \frac{i\lambda}{2} \int d_n k \int_0^1 dx \frac{1}{(k^2 - \xi)^2} = \frac{i\lambda}{2} \int_E d_n k \int_0^1 dx \frac{1}{(k^2 + \xi)^2}
\end{aligned} \tag{4.4}$$

where in the last equalities I have assumed that the integral is well defined, exchanged order of integration, and shifted $k \rightarrow k - xp$; I then performed a Euclidean rotation, replacing $k^0 \rightarrow ik^0$ (the subscript E , which I will henceforth drop, indicates that the integral is over Euclidean space).

The next step is to write the volume element in terms of polar coordinates $d_n k = k^{n-1} dk d\Omega_n / (2\pi)^n$ where $d\Omega_n$ is the element of solid angle in n dimensions. In practically all applications the only property of $d\Omega_n$ which is needed is its integral over the whole of the sphere in n dimensions: $\int d\Omega_n = 2\pi^{n/2} / \Gamma(n/2)$. Then I get

$$I_n = \frac{i\lambda\pi^{n/2}}{(2\pi)^n \Gamma(n/2)} \int_0^\infty dk \int_0^1 dx \frac{k^{n-1}}{(k^2 + \xi)} = \frac{i\lambda}{2} \frac{\Gamma(2 - n/2) \pi^{n/2}}{(2\pi)^n} \int_0^1 dx \xi^{(n-4)/2} \tag{4.5}$$

In this expression n is understood to be an arbitrary complex number, the only restriction being that the integral is well defined; for a discussion on the validity of this assumption see Ref. 4.

The final step is to replace $n = 4 - 2\epsilon$ with the understanding that $\epsilon \rightarrow 0$. Using $\Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma_E + O(\epsilon)$ where γ_E denotes Euler's constant, yields

$$I_n = \frac{i\lambda}{32\pi^2} \int_0^1 dx \left\{ \underbrace{\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) - \ln \xi}_{=C_{UV}} + O(\epsilon) \right\}. \tag{4.6}$$

The presence of a logarithm of a dimension-full quantity might seem a puzzling result of the method. Its origin can be traced back to the fact that in n dimensions λ is not dimensionless: the kinetic energy of the scalars fixes the canonical

dimension of the field at $n/2 - 1$, so that $d^n x \phi^4$ has dimensions $n - 4 = -2\epsilon$. It is therefore convenient to replace $\lambda \rightarrow \lambda \kappa^{2\epsilon}$ where κ is an arbitrary scale and λ is now dimensionless. The arbitrariness in the choice of κ is far from being a defect of the theory or the regularization method: the dependence of Green's functions on κ determines the renormalization group flow of the effective couplings; all observables are independent of this scale. With this replacement I get

$$I_n = \frac{i\lambda}{32\pi^2} \left\{ C_{UV} - \int_0^1 dx \ln(\xi/\kappa^2) + O(\epsilon) \right\}. \quad (4.7)$$

In a similar manner one can derive

$$\mathcal{J}_{km} = \int \frac{d^n \ell}{(2\pi)^n} \frac{(\ell^2)^k}{(\ell^2 - \xi)^m} = \frac{(-)^{k+m} i}{(4\pi)^{n/2}} \xi^{k-m+n/2} \frac{\Gamma(k + \frac{n}{2}) \Gamma(m - k - \frac{n}{2})}{\Gamma(m)} \quad (4.8)$$

It can be verified that $(\lambda/2)\mathcal{J}_{02} = I_n$.

In dimensional regularization we also have

$$\int \frac{d^n k}{(k^2)^\ell} = 0 \quad (4.9)$$

for all values of ℓ and n . This is a definition whose justification can be found in many of the excellent reviews on the subject⁴.

Similarly I can evaluate the graph

and obtain

$$\frac{1}{2}(-i\lambda) \int \frac{d^n \ell}{(2\pi)^n} \frac{i}{\ell - m^2 + i\epsilon} = \frac{\lambda}{2} \mathcal{J}_{01} = \frac{i\lambda}{32\pi^2} m^2 [C_{UV} + 1 + \ln m^2] \quad (4.10).$$

Note that this results is proportional to m^2 ; had I used a cutoff terms $\propto \Lambda_{UV}^2$ would also appear.

As advertised the results have divergences appearing as poles in $\epsilon = (4 - n)/2$. The fact that the corrections to the mass in (4.10) are $\propto m^2$ is a result of (4.9). Collecting all one loop contributions I finally get

$$= \frac{3i\lambda^2}{32\pi^2} C_{UV} + \text{finite}$$

$$= \frac{i\lambda}{32\pi^2} m^2 + \text{finite}$$

With these preliminaries I can give a bird's eye view of the renormalization program. Consider the corrections to the propagator. It is clear that we can separate graph as those that can be separated by cutting one internal line (called one particle reducible or 1PR graphs) and those that cannot (called one particle irreducible or 1PI graphs). Denoting the sum of all 1PI graphs by a grey disk, which I define to be the object $i\Pi$ below, I have the following relation

$$i\Pi$$

$$= \frac{i}{p^2 - m^2 + \Pi}$$

and this implies that the physical mass m_{phys} is determined by the relation

$$m_{\text{phys}} = m^2 - \Pi(p^2 = m_{\text{phys}}^2) \quad (4.11)$$

so that in the vicinity of m_{phys} ,

$$\frac{1}{p^2 - m^2 + \Pi} \simeq \frac{Z}{p^2 - m_{\text{phys}}^2}; \quad p^2 \simeq m_{\text{phys}}^2 \quad (4.12)$$

Similarly we have

I define the physical coupling constant as $\lambda_{\text{phys}} = \Lambda(\bar{p}, \bar{q})$ where $\bar{p} = \bar{q} = (2m, 0, 0, 0)$

To one loop I get

$$\begin{aligned} \lambda_{\text{phys}} &= \lambda - \frac{3\lambda^2}{32\pi^2} \left[C_{UV} + 2 - \ln \frac{m^2}{\kappa^2} \right] \\ m_{\text{phys}} &= m^2 \left[1 + \frac{\lambda}{32\pi^2} \left(C_{UV} + 1 - \ln \frac{m^2}{\kappa^2} \right) \right]. \end{aligned} \quad (4.13)$$

Taking λ_{phys} and m_{phys} as the input data the lagrangian parameters

$$\begin{aligned} m^2 &= m_{\text{phys}}^2 \left[1 - \frac{\lambda_{\text{phys}}}{32\pi^2} \left(C_{UV} + 1 - \ln \frac{m_{\text{phys}}^2}{\kappa^2} \right) \right] \\ \lambda &= \lambda_{\text{phys}} \left[1 + \frac{3\lambda_{\text{phys}}^2}{32\pi^2} \left(C_{UV} + 2 - \ln \frac{m_{\text{phys}}^2}{\kappa^2} \right) \right] \end{aligned} \quad (4.14)$$

and the important property of renormalization theory is that with these replacements, and with an appropriate rescaling of the field $\phi \rightarrow Z^{-1/2}\phi$, all infinities disappear (this is true to all orders in perturbation theory).

It is worth stopping for a minute at this point and determine a simple way of obtaining the relationship between the $1/\epsilon$ poles and the divergences in terms of the cutoff Λ . To do this note the estimate

$$\int \frac{d^n \ell}{(2\pi)^n} \frac{1}{(\ell^2 - m^2)^k} \propto (m^2)^{-k+n/2} \quad (4.15)$$

in dimensional regularization. With an UV cutoff in contrast this integral is $\propto \Lambda^{n-2k}$ if $n > 2k$, so that all but the logarithmic divergences “disappear” under dimensional regularization (which fact can be traced back to the property (4.9)). Note however that a quadratic divergence when $n = 4$ becomes a logarithmic divergence when $n = 2$ so that we can extract quadratic divergences from dimensional regularization by assuming $n = 2 - 2\epsilon$.

The four-point function has only a logarithmic divergence at one loop; this corresponds to the $1/\epsilon$ pole (where $2\epsilon = 4 - n$). The two point function has both quadratic and logarithmic divergences, the latter was obtained above. For the former note that when $n = 2 - 2\epsilon$ I have

$$\frac{\lambda}{2} \mathcal{J}_{01} \Big|_{n=2-2\epsilon} = -\frac{i\lambda}{32\pi^2} \left(C_{UV} - \ln \frac{m^2}{\kappa^2} \right)$$

In terms of an UV cutoff this same integral when $n = 4$ yields $-i\lambda\Lambda^2/(32\pi^2) + [i\lambda/(32\pi^2)]m^2 \ln(\Lambda^2/m^2)$ so we can identify C_{UV} with Λ^2 when $n \simeq 2$ (up to possible logarithmic and constant corrections) and with $\ln \Lambda^2$ when $n \simeq 4$ (up to constant corrections).

The main advantage of dimensional regularization is that it preserves many important symmetries, such as gauge invariance. It does present a problem when confronted with chiral interactions which involve γ_5 fermion couplings. The reason is that in four dimensions $\gamma_5 = \epsilon^{\mu\nu\alpha\beta} \gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta$ (up to a normalization constant) and

the extension to n dimensions is problematic since the ϵ tensor is peculiar to four-dimensional space time. The prescription that works⁴ is to define $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$ which satisfies

$$\{\gamma_\mu, \gamma_5\} = 0 \quad \mu = 0, 1, 2, 3; \quad [\gamma_\mu, \gamma_5] = 0 \quad \mu \neq 0, 1, 2, 3 \quad (4.16)$$

4.2. Example.

I will now apply the above formalism to consider the radiative corrections in an effective theory. I will assume a weakly-coupled underlying physics and, for the low energy theory, I will take the same simple scalar theory with a set of higher dimensional operators added to it. To simplify matters I will assume a symmetry under $\phi \leftrightarrow -\phi$ so that the lowest dimensional operators have dimension six. The only possibilities are then (symbolically) $\phi^2\partial^4$, $\phi^4\partial^2$, ϕ^6 .

$\phi^2\partial^4$: Two operators of this type are $(\partial_\mu\Box\phi)\partial^\mu\phi$ or $(\partial_\mu\partial_\nu\phi)^2$, both of these are equivalent (up to a total derivative) to the operator $(\Box\phi)^2$ which is the remaining possibility. The classical equations of motion read $\Box\phi + m^2\phi + \lambda\phi^3/6 = 0$, so that I can replace

$$(\Box\phi)^2 \rightarrow m^4\phi^2 + \frac{1}{3}\lambda m^2\phi^4 + \frac{\lambda^2}{36}\phi^6 \quad (4.17)$$

$\phi^2\partial^2$: One possibility is $\phi^3\Box\phi$ which, again by the use of the equations of motion, is equivalent to $m^2\phi^4 + \lambda\phi^6/6$. The remaining possibility is $\phi^2(\partial_\mu\phi)^2$ which is equivalent, via the equations of motion and integration by parts, to $m^2\phi^4/3 + \lambda\phi^6/18$.

From these arguments it follows that, modulo a renormalization of m and λ , the only operator I need to consider is ϕ^6 :

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \frac{\alpha}{\Lambda^2}\frac{\phi^6}{6!}, \quad (4.18)$$

which is renormalizable to $O(\alpha)$.

The only new divergences generated by the dimension six operator

$$= -i \left[-\frac{\alpha}{32\pi^2} \frac{m^2}{\Lambda^2} C_{UV} + \text{finite} \right]$$

$$= -\frac{i\alpha\lambda}{\Lambda^2} \frac{C_{UV}}{32\pi^2} + \text{finite}$$

As in the previous case I can define the physical ϕ^6 coupling as

$$= \frac{i}{\Lambda^2} \alpha_{\text{phys}}$$

where all the space components of the external momenta are assumed to be zero.

Now I can study the renormalization of the model. The input parameters I choose are the physical mass and couplings; these are defined by (I ignore higher loop corrections)

$$\begin{aligned} m_{\text{phys}}^2 &= m^2 \left[1 + \frac{\lambda}{32\pi^2} C_{UV} + \text{finite} \right] \\ \lambda_{\text{phys}} &= \lambda - \frac{3\lambda^2}{32\pi^2} C_{UV} - \frac{\alpha}{32\pi^2} \frac{m^2}{\Lambda^2} C_{UV} + \text{finite} \\ \alpha_{\text{phys}} &= \alpha \left[1 - \frac{15\lambda}{32\pi^2} C_{UV} + \text{finite} \right] \end{aligned} \tag{4.19}$$

which can be inverted to read

$$\begin{aligned}
m^2 &= m_{\text{phys}}^2 \left[1 - \frac{\lambda_{\text{phys}}}{32\pi^2} C_{UV} + \text{finite} \right] \\
\lambda &= \lambda_{\text{phys}} + \frac{3\lambda_{\text{phys}}^2}{32\pi^2} C_{UV} + \frac{\alpha_{\text{phys}}}{32\pi^2} \frac{m_{\text{phys}}^2}{\Lambda^2} C_{UV} + \text{finite} \\
\alpha &= \alpha_{\text{phys}} \left[1 + \frac{15\lambda_{\text{phys}}}{32\pi^2} C_{UV} + \text{finite} \right]
\end{aligned} \tag{4.20}$$

As before it is easy to verify that the whole of the renormalization program works: to any number of loops (but keeping a single insertion of the dimension six operator!) the replacement of Lagrangian parameters by their physical counterparts renders all results finite. At two loops (and beyond) the only new effect is the appearance of a divergence that is cancelled by an unobservable rescaling of the field: $\phi \rightarrow \phi/\sqrt{Z}$ for some constant Z , defined in (4.12).

To $O(\alpha^2)$ the above statements are not valid. Consider for example the graph

is associated with the operator ϕ^8 ; it diverges, and it is of order α^2 . If one needs the $O(\alpha^2)$ corrections then the operators of dimension eight should be included in the effective lagrangian. Now the renormalization program can be carried out to any loop order provided the graphs are restricted to having two dimension-six operator insertions or one dimension-eight operator insertion.

4.3. Gauge theories.

As discussed above the effective lagrangian must be required to be gauge invariant. One can always chose the unitary gauge, but in this case the propagator is, for a massive gauge boson (the mass is understood to be a result of spontaneously symmetry breaking)

$$\frac{-i}{p^2 - m^2 + i\epsilon} \left(g_{\mu\nu} - \frac{1}{m^2} p_\mu p_\nu \right) \delta_{ab} \xrightarrow{p \rightarrow \infty} \frac{i}{m^2} \frac{p_\mu p_\nu}{p^2} \delta_{ab} \tag{4.21}$$

where p is the momentum in the propagator and a and b are gauge indices (if any). Since the unitary gauge propagator does not vanish at infinite momentum

the various graphs are more divergent than in other gauges, such as the Feynman gauge. Because of these complications this choice of gauge is not very good for doing loop calculations; a more convenient choice is the Feynman gauge propagator

$$\frac{-i}{p^2 - m^2 + i\epsilon} \delta_{ab}. \quad (4.22)$$

In this case, however, we pay the price of having to include the contributions of the unphysical scalars.

In the Feynman gauge one must also include the contributions from the Fadeev-Popov ghost and the question naturally arises as to whether I should also consider effective operators containing these fields. The answer to this is no: effective operators are produced by integrating out heavy degrees of freedom and are therefore independent of the gauge fixing procedure used in the light sector of the model, hence no dependence on the ghosts can appear in the effective operators.

Another remark is pertinent at this point. The effective action is constructed by the following two step procedure. Consider the full action S and denote the light fields by ϕ while the heavy fields are denoted by Φ . Then construct W using

$$e^{iW} = \int [d\phi][d\Phi] e^{iS + \int \phi j} \quad (4.23)$$

where j denotes the sources for the light fields. Define then $\frac{\delta W}{\delta \phi} = \varphi$ which is clearly the average of the light field ϕ in the presence of sources. Finally define the effective action as

$$\Gamma = W - \int j\varphi \quad (4.24)$$

The gauge in S can be fixed so that Γ is a gauge invariant functional of the light fields φ ³⁴. Expanding Γ in powers of $1/\Lambda$ I obtain $\Gamma = \int \mathcal{L}_{\text{eff}} d^4x$. It follows that \mathcal{L}_{eff} is gauge invariant and will not contain the ghost fields generated when fixing the gauge for the light degrees of freedom.

As a specific loop calculation involving a gauge theory consider the contribution of \mathcal{O}_{WB} to the anomalous magnetic moment of the muon³⁵. The relevant vertices are

$$\begin{aligned} \mathcal{O}_{WB} &= \left(\phi^\dagger \sigma_I \phi \right) W_{\mu\nu}^I B^{\mu\nu} \\ &\rightarrow -2ig \frac{v^2}{2} c_w W_\mu^- W_\nu^+ F^{\mu\nu} + \sqrt{8} c_w \phi_0 F^{\mu\nu} [\phi_- \partial_\mu W_\nu^+ + \text{herm. conj.}] \end{aligned} \quad (4.25)$$

Its contribution appears in the following tree graphs;

these yield

$$\frac{m_\mu g c_w \alpha_{WB}}{16\pi^2 \Lambda^2} \left(C_{UV} + \frac{3}{2} - \ln \frac{m_w^2}{\kappa^2} \right) \sigma^{\alpha\beta} k_\beta + \dots \quad (4.26)$$

which implies that

$$a_\mu = \frac{m_\mu^2 \alpha_{WB}}{6\pi^2 t_w \Lambda^2} \left(C_{UV} + \frac{3}{2} - \ln \frac{m_w^2}{\kappa^2} \right) \quad (4.27)$$

The infinite contribution renormalizes the coefficients of the effective operators $\bar{\psi}_\mu \sigma_{\alpha\beta} \psi_\mu B^{\alpha\beta}$ and $\bar{\psi}_\mu \sigma^{\alpha\beta} \sigma_I \psi_\mu W_{\alpha\beta}^I$ which also generate a tree-level contribution to a_μ ³⁵.

5. Application potpourri

When dealing with the practical applications of the effective lagrangian formalism to electroweak processes there are two cases of interest: the decoupling case and the chiral case. In the first there is a Higgs excitation in the light theory and the effective operators are arranged according to their canonical dimension or, equivalently, to the power of Λ in their prefactor. For the chiral case there are no physical scalar excitations, and the terms in the effective lagrangian are ordered according to the number of derivatives in the effective operators.

For the decoupling case I will assume that the underlying physics is weakly coupled. This is associated with the fact that it is very difficult to maintain a light Higgs when the underlying physics is strongly coupled; this either requires fine tuning or that the low energy particle content be radically altered²⁷.

5.1. Four fermion operators.

There are two types of four-fermion operators which I will consider. The first one is of the form $(\bar{\psi}_1\gamma_\mu\psi_2)(\bar{\psi}_3\gamma^\mu\psi_4)$, while the second type is of the form $(\bar{\psi}_1\psi_2)(\bar{\psi}_3\psi_4)$. All other possibilities are equivalent via Fierz transformations (all possible chiralities are understood to be included). The coefficient of these operators can be written in the form g_H^2/Λ^2 .

For the decoupling case these operators are generated by the graphs

where each vertex factor is g_H and the mass of the exchanged particle is Λ . A simple realization is the low energy lagrangian for a model containing an extra neutral Z' vector boson. In this situation $g_H \lesssim 1$.

For the chiral case the coefficients of the operators are estimated using naive dimensional analysis. The same arguments used previously give $(g_H/\Lambda)^2 \sim \Lambda^4\lambda/\Lambda_\psi^6$ which, using $\Lambda \sim (4\pi)^{2/3}\Lambda_\psi$ and $16\pi^2\lambda \sim 1$, imply

$$g_H \sim 2\sqrt{\pi} \quad (5.1)$$

In order to translate a limit for Λ obtained in the chiral case to a limit for the decoupling case one only needs to multiply by $1/\sqrt{4\pi} \sim 0.3$. Thus in the decoupling case the sensitivity to new physics of any experiment is significantly degraded.

For the chiral case LHC will be sensitive to scales below 10 TeV^{36} using the p_T distribution of jets. A better bound is obtained by studying dilepton production³⁶

$$\Lambda \lesssim 15 \text{ TeV}; \quad \text{LHC (chiral case)} \quad (5.2)$$

A similar investigation for HERA yields³⁷

$$\Lambda \lesssim 1 \text{ TeV}; \quad \text{HERA (chiral case)} \quad (5.3)$$

For the decoupling case this last estimate translates into $\Lambda \lesssim 300 \text{ GeV}$ which is consistent with the previous sensitivity limit obtained using helicity violating processes.

5.2. Triple vector-boson couplings.

One set of effective couplings which have been extensively studied describe the interactions between the W bosons and the photon or the Z . These are usually presented based on an effective lagrangian whose only explicit symmetry is Lorentz and electromagnetic gauge invariances. The by now standard representation is³⁸

$$\begin{aligned}
\mathcal{L}_{WWV}/g_{WWV} = & ig_1^V \left(W_{\mu\nu}^\dagger W^\mu V^\nu - \text{h.c.} \right) - g_4^V W_\mu^\dagger W_\nu (\partial^\mu V^\nu + \partial^\nu V^\mu) \\
& + i\kappa_V W_\mu^\dagger W_\nu V^{\mu\nu} + i\frac{\lambda_V}{m_w^2} W_{\lambda\mu}^\dagger W^\mu{}_\nu V^{\nu\lambda} \\
& + i\tilde{\kappa}_V W_\mu^\dagger W_\nu \tilde{V}^{\mu\nu} + i\frac{\tilde{\lambda}_V}{m_w^2} W_{\lambda\mu}^\dagger W^\mu{}_\nu \tilde{V}^{\mu\nu} \\
& + g_5^V \epsilon^{\mu\nu\rho\sigma} \left(W_\mu \overset{\leftrightarrow}{\partial}_\rho W_\nu \right) V_\sigma
\end{aligned} \tag{5.4}$$

(where terms proportional to $\partial \cdot W$ and $\partial \cdot V$ are ignored). V denotes either the photon or the Z field; in the first case only terms in compliance with electromagnetic gauge invariance are retained. It is assumed (without loss of generality) that $g_{WW\gamma} = -e$, $g_{WWZ} = -e \cot \theta_w$. All field tensors are understood to be Abelian, for example, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$.

For the chiral case the magnitudes of the coefficients are estimated using naive dimensional analysis. The results are

$$\begin{aligned}
|g_i^V|, |\kappa_V|, |\tilde{\kappa}_V| & \sim \frac{g^2}{16\pi^2} \sim 3 \times 10^{-3} \\
|\lambda_V|, |\tilde{\lambda}_V| & \sim \frac{m_w^2}{\Lambda^2} \frac{g^2}{16\pi^2} \sim 2 \times 10^{-6} \left(\frac{4\pi v}{\Lambda} \right)^2
\end{aligned} \tag{5.5}$$

These estimates are modified should there be resonances of masses $\sim 4\pi v$. In this case the presence of long tails could enhance the above values by a factor of $\lesssim 10$. This then implies

$$\begin{aligned}
|g_i^V|, |\kappa_V|, |\tilde{\kappa}_V| & \lesssim 10^{-2} \\
|\lambda_V|, |\tilde{\lambda}_V| & \lesssim 10^{-5} \left(\frac{4\pi v}{\Lambda} \right)^2
\end{aligned} \tag{5.6}$$

Note also that for the chiral case the λ terms are of the form $(\partial W)^3$ corresponding to six covariant derivatives (recall that a field tensor equals a commutator of covariant derivatives). These terms are then subdominant.

For the decoupling case all coefficient are $\sim (v/4\pi\Lambda)^2$ (with a possible enhancement of $\lesssim 10$). It then follows that implies

$$|g_i^V|, |\kappa_V|, |\tilde{\kappa}_V|, |\lambda_V|, |\tilde{\lambda}_V| \lesssim \left(\frac{2 \times 10^{-3}}{\Lambda_{\text{TeV}}^2} \right)^2 \quad (5.7)$$

Consider now the application of this lagrangian to tree level processes.

HERA

In this case the best bound obtained is $|\kappa - 1| \lesssim 0.3$ ³⁹ using the reaction $ep \rightarrow \nu\gamma X$ and assuming five year's integrated luminosity.

Fermilab Tevatron

A similar investigation yields the constraints $|\kappa|, |\lambda| \lesssim 2$ ^{40,41} using W^+W^- production.

LEP2

For this accelerator a strong effort has been devoted to understanding the properties of the reaction $e^+e^- \rightarrow W^+W^-$ when the effects of (5.4) are included. When the cross section for this process is calculated, a term $\propto \lambda s^2/m_w^2$ is generated³⁸. This will generate enormous deviations from the Standard Model prediction provided one (erroneously) assumes $\lambda \sim 1$. If the natural order of magnitude for λ is used all such effects constitute small corrections to the Standard Model amplitude. This is true for all effects derived from (5.4).

Not all corrections are negligible. Consider for example the presence of a particle or particles that guarantee a unitary cross section. Suppose also that the CM energy of the collider under consideration is below the mass(es) of such particle(s). Then the unitarity cancellations generated by these excitations are only partially effective and we can get a glimpse at their presence. This in fact is verified by explicit calculation for the corrections to the reaction under consideration generated by a heavy fourth family. The cross section acquires a correction factor of $\sim 1 + (s/m_w)^2(m_w/4\pi v)^2$ which can reach 10%. Note that it would be wrong to state that the corrections are $O(s/m_w^2)$ since these are enormous; one must remember that this factor is accompanied by a small coupling.

I would like to point out that when calculating the reaction considered in this subsection it is common practice to include only the diagrams generated by a Z or γ s -channel exchange. There are however other graphs such as

generated by the Standard Model and by five dimension-six operators. Finally there are contact terms generated by the operator

$$i(\bar{\ell}\sigma_I\gamma^\mu D^\nu\ell)W_{\mu\nu}^I = \frac{1}{2}g(\bar{e}_L\gamma^\mu e_L)W_{\mu\nu}^-W^{+\nu} + \dots \quad (5.8)$$

If we assume that the order of magnitude of the neglected graphs is similar to the one obtained from the contributions generated by \mathcal{L}_{WWV} then the graphs which are not included would not alter the estimate on the sensitivity to the scale of new physics. This however, constitutes an added assumption on the model and should be kept in mind when evaluating the constraints on Λ derived from various calculations.

LHC

Various investigations^{42,43,44} have considered limits on the coefficients of (5.4) and derive the sensitivity limits $|\kappa|, |\lambda| \lesssim 0.3$ or $|\kappa|, |\lambda| \lesssim 0.1$ which are obtained by considering final states with only two vector bosons, W^+W^- , ZZ , ZW^\pm , $W^\pm W^\pm$. The most promising of these is the ZZ final state. The idea is then to measure the p_T distribution of the Z vectors and search for a deviation from the Standard Model predictions.

Note however that LHC will probe energies above 3 TeV where the non-linear realization of the symmetry (chiral case) is not applicable. In order to deal with this problem one can simply impose cuts that insure that this bound is not violated⁴³; this of course degrades the signal to noise ratio. If no sever cuts are imposed, then a recipe for extending the model to energies above $4\pi v$ must be provided. This is often done by postulating the appearance of new resonances (see Ref. 45 and references therein). Of these two possibilities the first one preserves all the features of the effective lagrangian approach, the second one is, by its very nature, model dependent.

NLC

For this accelerator, assuming $\sqrt{s} = 1.5$ TeV and a luminosity of 100/fb, the limits on the coefficients are significantly improved: $|\kappa| \sim 10^{-2} - 10^{-3}$ ^{44,42}. For

the chiral case one must remember that this accelerator is close to the limit of validity of the effective lagrangian parametrization, in fact $\sqrt{s} \sim (4\pi v)/2$ so that corrections of the order of $\sim (1/2)^2 \sim 25\%$ can be expected. When $\sqrt{s} = 500$ GeV CM energy similar limits on κ are obtained.

For the chiral case one can also derive bounds on the coefficients of

$$\mathcal{L} = \frac{c_L g}{16\pi^2} \text{tr} \left\{ \mathbf{W}_{\mu\nu} (D^\mu U)^\dagger (D^\nu U) \right\} + \frac{c_R g'}{16\pi^2} \text{tr} \left\{ B_{\mu\nu} \sigma_3 (D^\mu U)^\dagger (D^\nu U) \right\} \quad (5.9)$$

where we expect $|c_{L,R}| \sim 1$. The results are⁴²

Limits on $ c_{L,R} $ (chiral case)				
Coupling	Experiment (future)			
	LEP2	NLC (1)	NLC (2)	LHC
$ c_L $	30	2	1	15
$ c_R $	150	10	5	150

where the sensitivity of other accelerators are included for comparison. The luminosity of LEP2 is assumed to be 500/pb and NLC (1) denotes a 500 GeV machine at 10/fb; NLC (2) denotes a 1 TeV machine at 44/fb. Finally, the LHC is assumed to have a luminosity of 10/fb. Note that a machine such as the NLC will have the necessary sensitivity to probe the region where the couplings have their natural sizes.

The sensitivity to λ is also calculated⁴²; the result being $|\lambda| \lesssim 0.01$.

$e\gamma$ and $\gamma\gamma$ colliders.

When a laser photon is backscattered in an e^+e^- collider several observational windows are opened up. The process $e\gamma \rightarrow ZZ e$ can be used to measure κ to a precision of 0.1, similarly $\gamma\gamma \rightarrow WW, ZZ$ can also measure κ to a precision of 0.01^{46,47}.

$b \rightarrow s\gamma$.

This is a one loop process if flavor mixing is ignored in the dimension-six operators. Form the CLEO bound⁴⁸ of $B(B \rightarrow K^* \gamma) = (4.5 \pm 1.4 \pm 0.9) \times 10^{-5}$ one can derive $B(b \rightarrow s\gamma) < 5.4 \times 10^{-4}$ which translates into $|\kappa|, |\lambda| \lesssim 10^{49,50,47}$.

AGS851

The one loop contributions stemming from \mathcal{L}_{WWV} to the muon anomalous magnetic moment can be calculated^{51,35}. In terms of the effective operator parametriza-

tion used in these lectures, the only contributions come from \mathcal{O}_{WB} and \mathcal{O}_W . Choosing the natural sizes for the coefficients the results are

$$\frac{g-2}{2} \sim \begin{cases} 10^{-12}/\Lambda_{\text{TeV}}^2 & (\text{decoupling case}) \\ 10^{-13} & (\text{chiral case}) \end{cases} \quad (5.10)$$

while the sensitivity of AGS851 is 10^{-10} ⁵²: the one loop contributions are completely negligible; the corresponding bounds for the coefficients of (5.4) are $O(10)$. In contrast the tree level contributions generate a (non-trivial) bound of ~ 700 GeV for Λ ³⁵.

5.3. Other operators.

In this section I give two examples of limits set on Λ using other operators. this list is far from exhaustive (for a more comprehensive compilation see Ref. 27).

S , T and U .

I have mentioned previously that these are important parameters to measure. For the chiral case we expect $S, U \sim 0.1$. In contrast, for the decoupling case, $S \simeq 0.3/\Lambda_{\text{TeV}}^2$ and $U \sim 0$ if $\Lambda \gtrsim 3$ TeV^{53,13,54,27}

Z widths, τ polarization, ν cross sections.

The sensitivity of the blind operators

$$\mathcal{O}_1 = B_{\mu\nu} (D^\mu \phi)^\dagger (D^\nu \phi); \quad \mathcal{O}_2 = W_{\mu\nu}^I (D^\mu \phi)^\dagger \sigma_I (D^\nu \phi); \quad (5.11)$$

to these processes can be calculated⁵⁵. With a natural size for the coefficients, the sensitivity to Λ is quite poor: $\Lambda \gtrsim 20$ GeV for LEP1 $\Lambda \gtrsim 100$ GeV for LEP2.

5.4. Various.

Heretofore I assumed the effective lagrangian to have the same symmetry structure as the Standard Model. In this section I will describe two instances where this assumption is modified.

$SU(2)_R$

Up to now the only symmetries imposed on the effective lagrangian were those of the Standard Model. One can of course modify this. For example, one can assume that the underlying physics conserves $SU(2)_R$ ⁵⁶, the approximate symmetry of the scalar sector. Even though the kind of reactions where the new physics effects are most noticeable are quite peculiar in this case, one finds no measurable effect on LEP2 once the natural size for the coefficients is used.

Extended groups.

One need not require the effective lagrangian to be invariant under the same group as the Standard Model. One may assume, for example, the presence of an extra $U(1)$ factor, and that this extended group is broken to the Standard Model group at a scale relatively close to the Fermi scale⁵⁷. Since the gauge group is larger, there are more constraints on the operators; on the other hand, since the particle content is increased, there are more operators that can be constructed. Of these opposing tendencies the second one dominates: there will be many more operators when the gauge group is increased.

In this scenario there is an additional gauge boson B' associated with the new $U(1)$ factor, and one can consider the effects of the operator

$$\mathcal{O}_{WB'} = \frac{\epsilon}{v^2} B'_{\mu\nu} W^{I\mu\nu} \left(\phi^\dagger \sigma_I \phi \right) \quad (5.12)$$

on a variety of LEP2 observables. Using W pair production significant effects are found for the choices $\epsilon = -0.2$, $m_{Z''} = 300$ GeV and when the new gauge coupling g'' is 0.067. Note however that the natural size for ϵ is 0.04 for which value the effects of this operator on LEP2 processes is negligible.

6. Conclusions

- Effective lagrangians constitute a coherent model and process independent scheme in which to study all possible effects from physics beyond the Standard Model. The whole approach is general and consistent.
- It has been unfortunately the case that the effective lagrangian approach has been often misused. For example, it is not uncommon to find striking claims on the sensitivity of a given experiment to Λ only to discover later that the calculations do not respect the restriction $\sqrt{s} < \Lambda$.
- When used consistently they provide good, (though not spectacular) bounds on the scale of new physics: $\Lambda \gtrsim 500$ GeV (from HERA and the muon anomalous magnetic moment); and $\Lambda \gtrsim 15$ TeV from the four-fermi operator effects at LHC.
- One can certainly use them in perturbative calculations. In a consistent scheme, however, these radiative corrections are almost always negligibly small (due to the present precision attained in the experiments).
- Given the untimely demise of the SSC and the probable postponement of the LHC this is a very good field in which to work given that direct evidence of new physics might be a decade (or longer) away.

Acknowledgements: The author would like to thank the organizers for the invitation to this course. I also would like to thank M. Einhorn, C. Arzt, M.-A. Perez and J. Toscano for very many illuminating discussions. This work was supported in part through funds provided by the DoE and the SSC Laboratory.

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